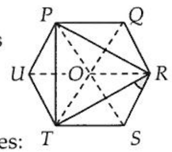
16 B 50 ml — The ratio 7 : 4 is equivalent to 350 : 200. Since the quantity of spit remains at 350 ml, the quantity of venom must increase to 200 ml, an additional 50 ml.

17 D 30° — There are many ways to consider this problem: one can see the regular hexagon *PQRSTU* as divided into six equilateral triangles joined at the centre *O*. So angle $RST = 2 \times 60° = 120°$. Since triangle RST is isosceles, angle $SRT = (180 - 120)° \div 2 = 30°$.

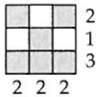


18 E 6 cm — The table below shows the values of the surface areas and volumes:

side length (cm)	2	3	4	5	6
surface area (cm^2)	24	54	96	150	216
volume (cm^3)	8	27	64	125	216

19 E 1 000 000 — Since 1 m = 100 cm, 1 m^2 = 100×100 cm^2 = 10 000 cm^2, and so 50 m^2 = 500 000 cm^2. With two daisies for every square centimetre, Daisy will plant 1 000 000 daisies.

20 E

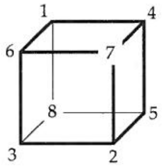


Four of the diagrams can be eliminated as follows: diagram A requires the rightmost column to have no shaded squares, while the top row has all squares shaded – this is impossible; in diagram B, the total number of shaded squares in the three rows is less than the total in the three columns, again impossible; diagram C exhibits a problem similar to that of diagram A; diagram D apparently requires four of the three squares in the bottom row to be shaded, which makes no sense. Only diagram E is possible, as is shown on the left.

21 D 18 — If this were possible, the top and bottom faces would, between them, include all the numbers from 1 to 8, and have no vertex in common. Therefore they would have a combined total of

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36.$$

Having the same 'face total', each of these two faces would have to add up to half of 36, ie. 18. The diagram on the right shows one way of achieving this.



22 C 43p — Given that 1 lemon costs the same as 3 plums, Chloe could have bought 10 plums and 2 oranges for £1, and Ginny 6 plums and 2 oranges for 72p. From this we can deduce that the cost of 4 plums is £1 – 72p = 28p, meaning that 1 plum costs 7p. Therefore 1 lemon costs 3×7p = 21p, and 2 oranges cost $(72 - 6 \times 7)$p = 30p. Thus the cost of one of each fruit is $(21 + 15 + 7)$p = 43p.

23 A 75 — Given that the product, 15 600, is a multiple of 25, the three numbers between them must contribute two factors of 5; since they are consecutive and there are only three of them, this can happen only if one number is itself a multiple of 25. It is now worth observing that the product of three consecutive numbers is roughly the same as the cube of the middle of the three. Since $20^3 < 15\,600 < 30^3$, the middle number must lie between 20 and 30, hence one of the numbers must be 25. The numbers can therefore be {23, 24, 25}, {24, 25, 26} or {25, 26, 27}. The product $15\,600 = 13 \times 1200$, so it has factors of both 4 and 13. The triple {25, 26, 27} has no factor of 4, and {23, 24, 25} no factor of 13. So, by elimination, the numbers are 24, 25 and 26, and their total is 75.

24 C 9 — The table on the right shows all the ways in which it is possible for every child to get the wrong scarf.

B	J	L	P
J	B	P	L
J	P	B	L
J	L	P	B
L	B	P	J
L	P	B	J
L	P	J	B
P	B	J	L
P	L	B	J
P	L	J	B

25 D Devi — After Amit says "2015", there are 5102 – 2015 = 3087 numbers remaining. Thus the counting goes round the table $3087 \div 6 = 514$ times with a remainder of 3, so the last to count is Devi.

Some notes and possibilities for further problems

Q3 Hair colour does not, in fact, lead to very clearly defined groups. Children might attempt a survey of hair and see what difficulties they face! Perhaps there are more suitable traits to use in a survey – this might lead to a useful discussion about the power and limitations of statistical analysis.

Q4 The question refers to the *comma butterfly* (cited in *RSPB News*, Spring 2014). The report of the Intergovernmental Panel on Climate Change in 2013 provides further data on species migration.

Q5 This question arose out of a visit to the Natural History Museum in London. Search out some comparable figures for other dinosaurs. How did a *giganotosaurus* or even a *spinosaurus* compare with *T. rex* in height and mass? And now compare them with a London double-decker bus!

Q6 Designing multipart machines is quite good fun: in this case what looks horrendous is rendered easy by multiplication by zero. One can also design machines that simply return whatever the input was originally; for example:

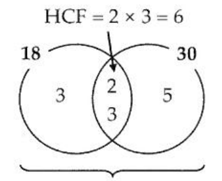
→ [× 3] → [+ 8] → [× 4] → [+ 4] → [÷ 2] → [– 18] → [÷ 6] →

Q7 With three dice, we could ask how many different products of the three numbers (like 90) are there? Perhaps there are not as many as one might think, since some are achievable in several ways: $24 = 1 \times 4 \times 6 = 2 \times 2 \times 6 = 2 \times 3 \times 4$.

Q8 It might be interesting to ask pupils what hints may help others to tackle this question efficiently.

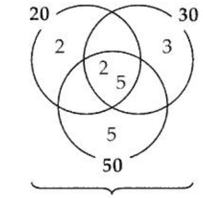
Q10 The method for representing the factors of several numbers in a Venn diagram may be well known – it allows one to calculate *highest common factor* (HCF) and *lowest common multiple* (LCM).

Here are 18 and 30 written as the product of prime factors:



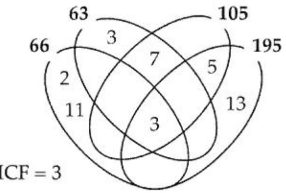
LCM = 3 × 2 × 3 × 5 = 90

Similarly with the numbers 20, 30 and 50 in this question:



LCM = 2 × 3 × 2 × 5 × 5 = 300

And with the four numbers 63, 66, 105 and 195:



HCF = 3

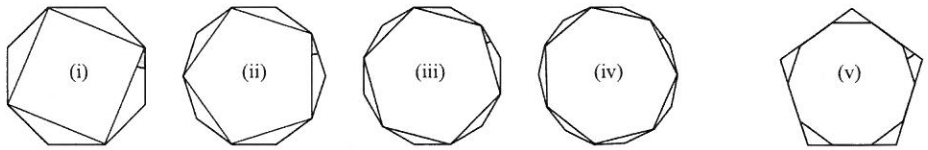
LCM = 2 × 3 × 3 × 5 × 7 × 11 × 13 = 90 090

Q11 There are many possible questions concerning the angles between the two hands of a clock – perhaps we should make the most of them while we can, before Big Ben has the only example of an analogue display! So:

- through what angle does the minute hand move in one minute?
- what is the acute angle between the hands at 6.30 pm?
- what is the obtuse angle between the hands at 12.20 pm? 10.24 am?
- for how many moments in a day do the hands overlap exactly?

Q12 How could children arrange the five cubes to minimise or maximise the visible surface area?

Q17 What about the angle between a square and an octagon (i), or a pentagon and a decagon (ii), or … ?



If the outer polygon has twice as many sides as the inner one, is there a pattern in your answers?

If the inner polygon has twice as many sides as the outer, as in diagram (v), does the angle differ from before? Is it larger or smaller now the polygons are 'reversed' and is there any connection?

Primary Mathematics Challenge – February 2015

Answers and Notes

These notes provide a brief look at how the problems can be solved.
There are sometimes many ways of approaching problems, and not all can be given here.
Suggestions for further work based on some of these problems are also provided.

P1 **E** we cannot tell   P2 **B** 8

1	**C**	7 kg	It should be a moment's work to see that Felix weighs 3 kg, and Marmalade 7 kg.
2	**E**	317°	The sum of the four angles of every quadrilateral is 360°, so the sum of the remaining three must be (360 – 43)° = 317°.
3	**E**	240	The percentage of brown-haired people is 100 – 11 – 23 – 36 = 30%. The number of brown-haired people is 30% of 800 = 30 × 8 = 240.
4	**E**	1.8 km	Moving 5 m for each of 365 days in a year gives a distance of 1825 m ≈ 1.8 km.
5	**C**	6400 kg	The options lead to the average mass as respectively 0.32 kg, 3.2 kg, 32 kg, 320 kg and 3200 kg; only the third, 32 kg represents a reasonable mass for a ten-year-old child.
6	**D**	56	After the fourth part of the machine any input up to that point will be multiplied by 0, and so will be 0; subsequent addition of 56 will give 56.
7	**B**	14	Given that the numbers on a die are 1, 2, 3, 4, 5 and 6, to achieve a product of 90, one of the numbers must be 5 and the other two must have a product of 18. Thus the numbers can only be 3, 5 and 6, whose sum is 14.
8	**A**	173	Trial and error should lead to the discovery that 56 + 25 + 37 = 118, which leaves the option 173 unused.
9	**A**	$\frac{1}{3}$	The rectangle has an area of 3 × 6 = 18 squares. The uppermost unshaded right-angled triangle has an area of half of the rectangle, ie. 9 squares, while the unshaded triangle in the bottom right-hand corner has an area of 3 squares. This leaves an area of 6 squares for the shaded triangle, one third of the rectangle.
10	**D**	10.05 am	The three girls all hiccup together after a number of seconds which is a common multiple of 20, 30 and 50. The *least common multiple* of these numbers is 300, so that they hiccup together after 300 seconds, ie. at 10.05 am.
11	**C**	4.30 pm	Each hour, the hour hand turns 30°, so it will take 5 ½ hours to turn 165°; the time will therefore be 4.30 pm.
12	**B**	17	Each of the five cubes has six faces. Five faces are touching the table and so not visible, and eight faces are joined in pairs; thus leaving visible 5 × 6 – 5 – 8 = 17.
13	**A**	90 sec	In order to nick Rob, Nick will have to make up the distance between them. His speed relative to Rob's is 4 km/h, or 1 km per 15 minutes. So to cover an extra 100 m he will take 1.5 minutes = 90 seconds.
14	**B**	25 cm²	The length of one side of the square is 10 cm, and so its area is 100 cm². The triangle forms one quarter of the square and so has an area of 25 cm².
15	**D**	9	Since 4 large eggs weigh the same as 6 medium eggs, 5 large eggs weigh the same as $6\times\frac{5}{4}$ medium eggs. Since 5 medium eggs weigh the same as 6 small eggs, $6\times\frac{5}{4}$ medium eggs weigh the same as $6\times\frac{5}{4}\times\frac{6}{5}$ small eggs. This is the same as 9 small eggs.

Q18 Pupils might extend the table given in the answers, and then discuss and investigate any patterns they notice. One curious result arises from multiplying the numbers for the surface area and the volume together and then dividing their product by 6 (hint: look at the units digit) – why is this?

Q21 Is the numbering of the vertices (given in the answers above) a unique solution, or are there others (symmetry excluded)?

Q22 For the purposes of the question, the information that 1 lemon costs the same as 3 plums is superfluous. Because, if we know that

3 lemons + 2 oranges + 1 plum cost 100p (*)
and 1 lemon + 2 oranges + 3 plums cost 72p,

then we can see that 2 lemons are 28p more expensive than 2 plums, or, more simply, that 1 lemon costs 14p more than 1 plum. If we now look again at the starred information and replace one of the lemons with one extra plum, we would know that

2 lemons + 2 oranges + 2 plums cost (100 – 14)p
and so 1 lemon + 1 orange + 1 plum cost ((100 – 14) ÷ 2)p = 43p.

It is somehow surprising that even though we can find out how much the three fruit cost in total, without extra information we cannot find out what each individual fruit costs.

Q23 Some points here for children to ponder (and prove) could include:

- the sum of 3 consecutive numbers is always 3 × the middle number
- the product of 3 consecutive numbers is always a multiple of 6, and ends in 0, 4 or 6
- the product of 4 consecutive numbers ends with 0 or 4
- the product of 5 or more consecutive numbers ends with 0.

Q24 This question has many manifestations: how many ways are there for a secretary to put letters in completely the wrong envelopes; or for a teacher to get a class of students to mark homework, without marking their own. The topic comes under the mathematical heading of *Derangements*, and mathematicians denote the number of ways of deranging n objects by $!n$ or (as here) D_n.

Firstly, it is obvious that $D_2 = 1$ (as two scarves can merely be swapped).
With three children and scarves, the only derangements are shown here, so $D_3 = 2$.

B	J	L
L	B	J
J	L	B

Above we found 9 ways for Bill, Jill, Lil and Phil all to get the wrong scarf, but how can we be sure that there are no other ways? One way to think about it is as follows:

let us presume (initially) that Bill takes Jill's hat – there are now two possibilities:

(a) Jill takes Bill's hat, or
(b) Jill doesn't take Bill's hat.

For possibility (a), there are D_2 ways in which Lil and Phil can each get the wrong scarf.
For possibility (b), there are D_3 ways in which the three children Jill, Lil and Phil can each get the wrong scarf of the three scarves remaining.

Initially Bill had three possibilities for the wrong scarf (not just Jill's), and so,

$$D_4 = 3 \times (D_3 + D_2) = 3 \times (2 + 1) = 9.$$

More generally, we can use the reasoning above to get the following formula:

$$D_{n+1} = n \times (D_n + D_{n-1}).$$

This means that five children have $D_5 = 4 \times (D_4 + D_3) = 4 \times (9 + 2) = 44$ ways of swapping scarves.
How many ways will Bill, Jill, Lil, Phil, Till and Will have of each getting a wrong scarf?

The PMC is organised by The Mathematical Association

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